Thermalization and localization in constrained disordered systems

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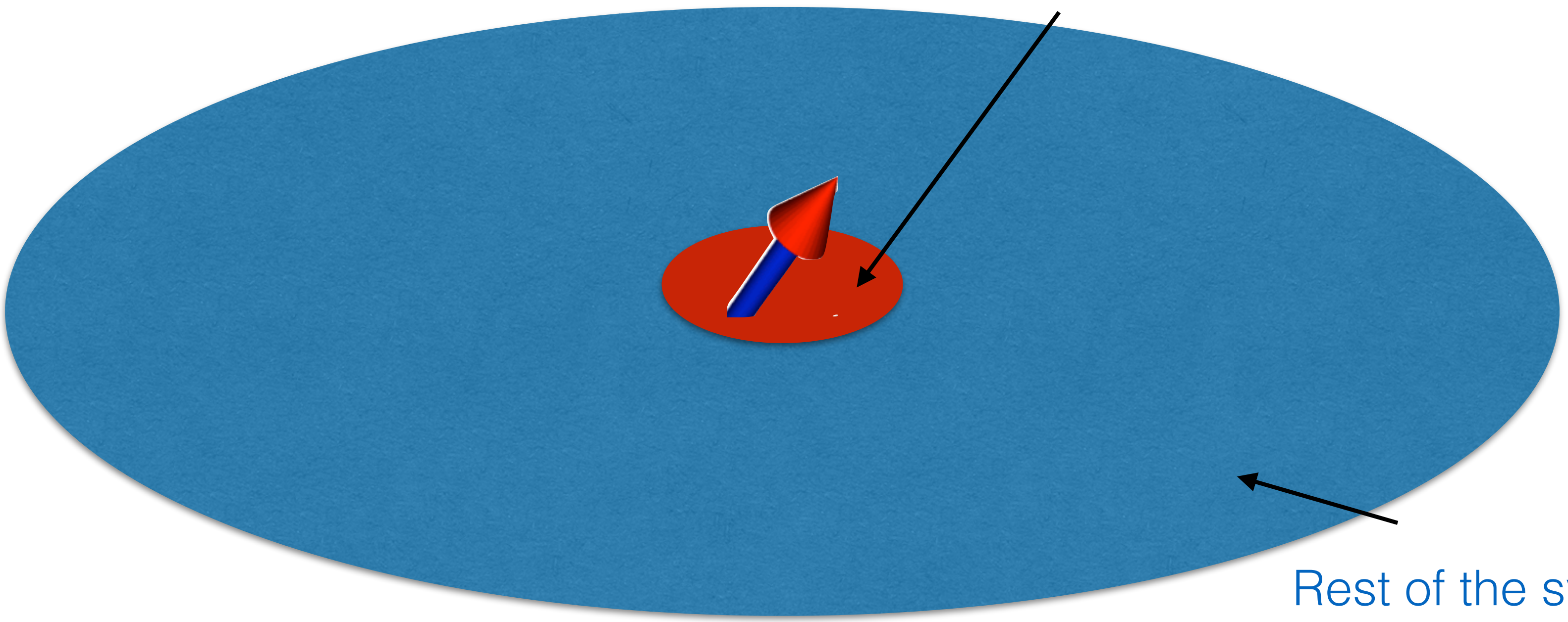
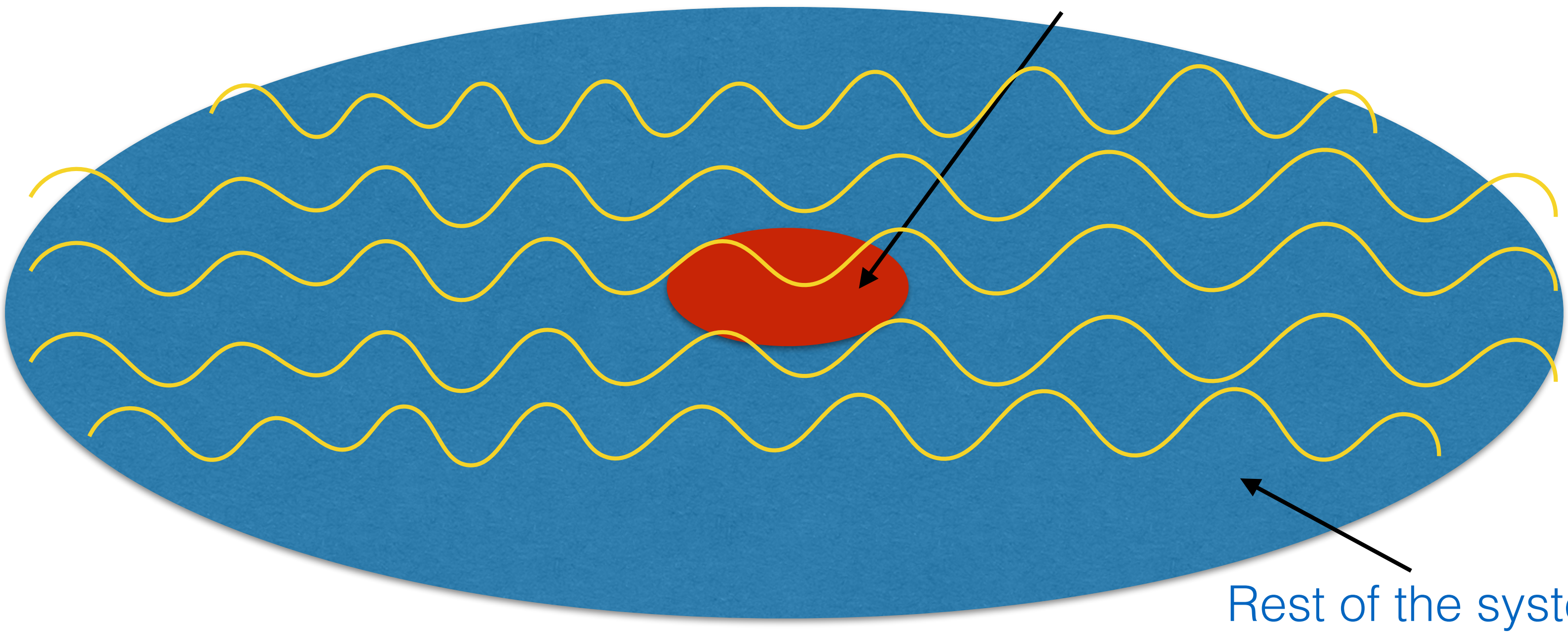
Quantum chaos, or thermalization of closed quantum systems



- Q: If you start in a state where all spins in A point up, how does the system thermalize?

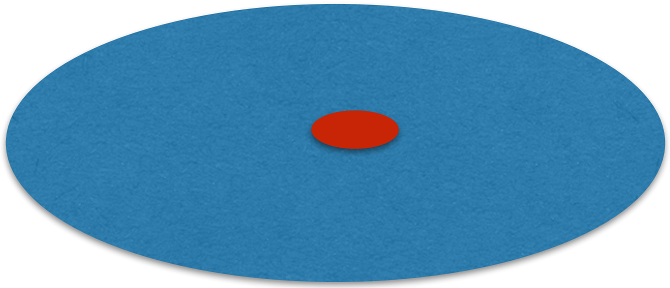
(How) do closed quantum systems thermalize?



- Information about this initial state cannot be lost, but it typically becomes highly delocalized
- So A looks thermal, because no information about the initial state can be recovered by looking only at A.

Eigenstate Thermalization Hypothesis (ETH)

(Deutsch and Srednicki ’91)



Temperature set by energy density

$$\langle \Psi_n | \hat{O} | \Psi_n \rangle = \frac{1}{Z_\beta} \mathrm{tr}\left(e^{-\beta H} \hat{O} \right) + \mathcal{O}\left(e^{-S(E_n)/2} \right)$$

Eigenstate (finite energy density)

Correction (finite size)

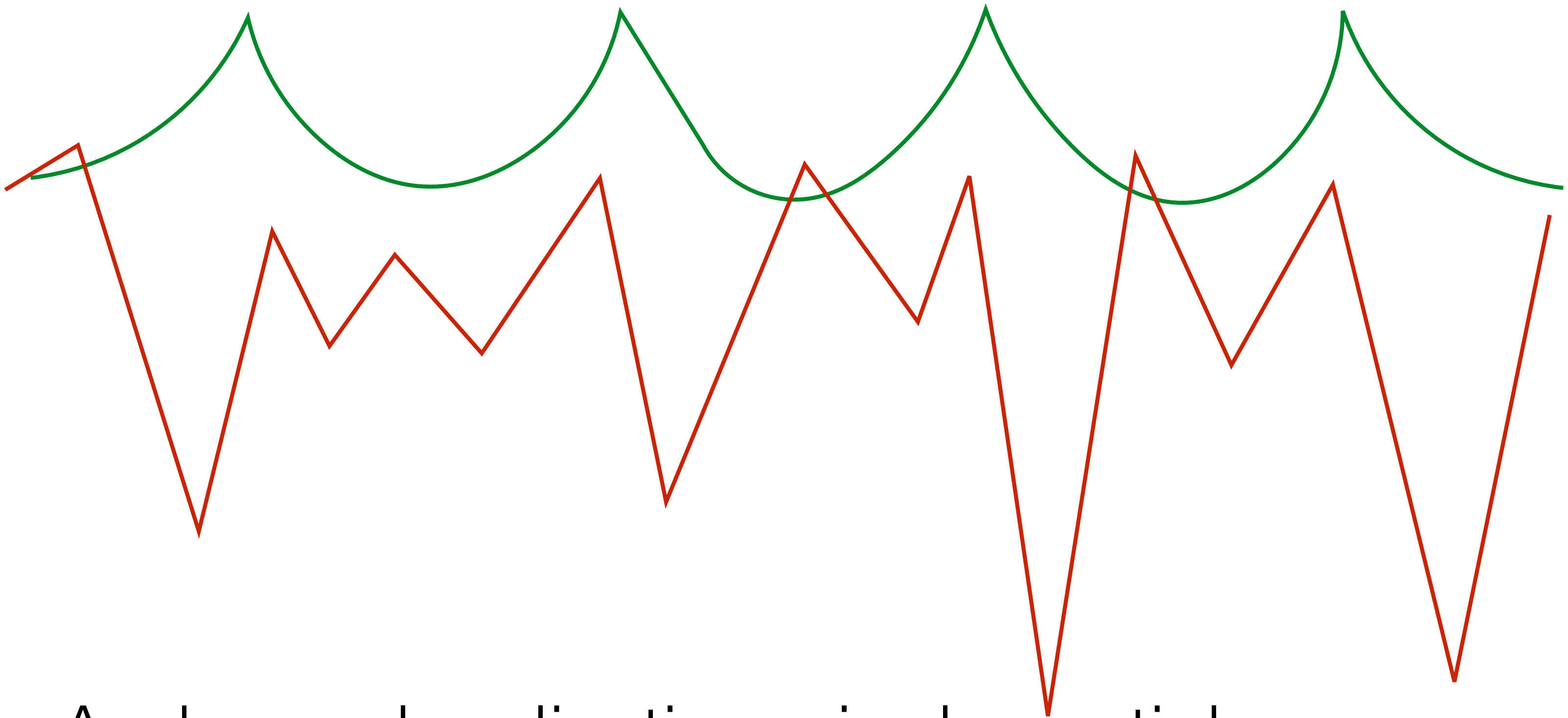
$$E_n = \frac{1}{Z_\beta} \mathrm{tr}\left(e^{-\beta H} H \right)$$

e.g. $S_A \propto V_A$

Eigenstate Thermalization Hypothesis (ETH)

- If true, closed quantum system thermalizes, and thermodynamics holds. (“Thermal phase”)
- But this is not the only option!

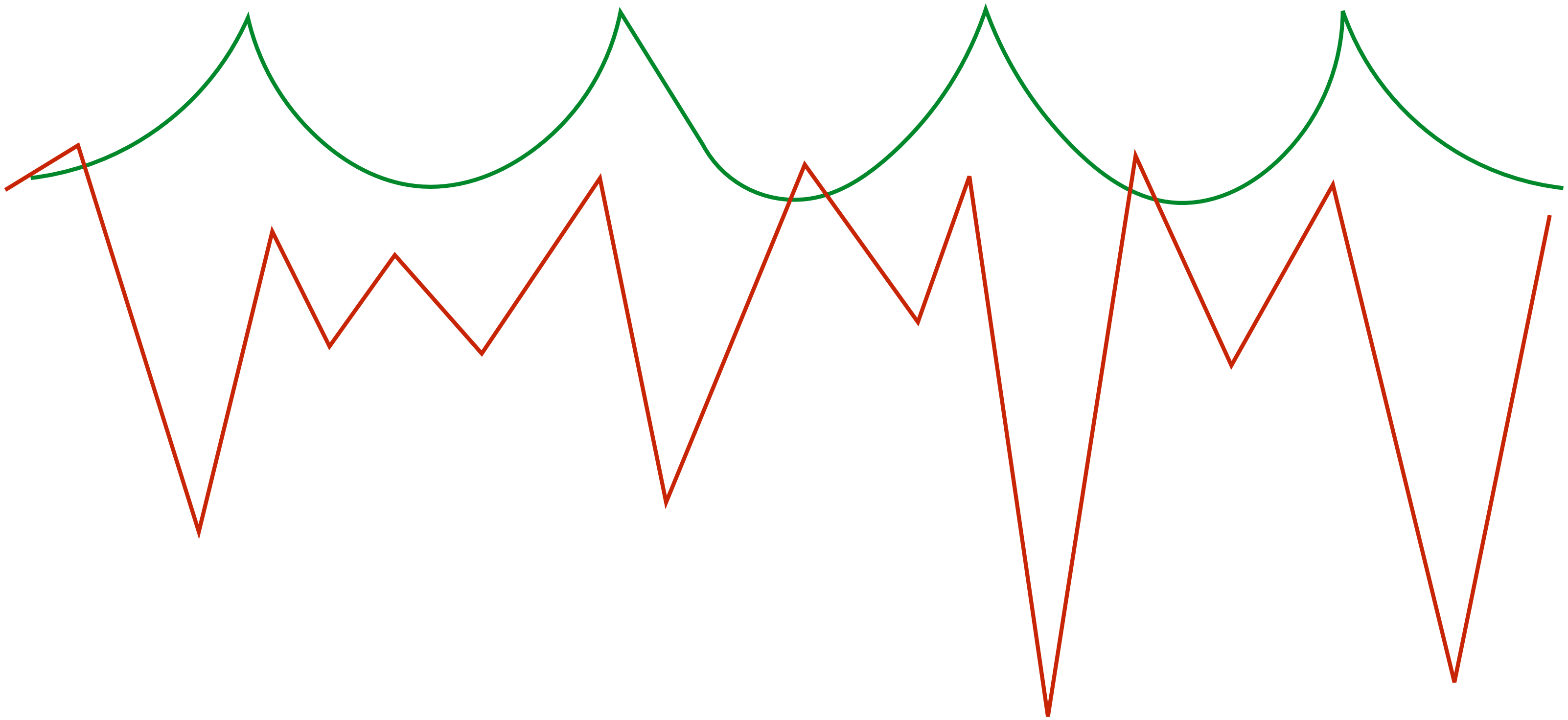
Localization



- Anderson localization: single-particle wave functions are exponentially localized in space.
- No transport, so an infinitely long local memory of initial state

Basko Aleiner Altshuler '05
Imbrie '16
and many others (Review: Nandkishore & Huse)

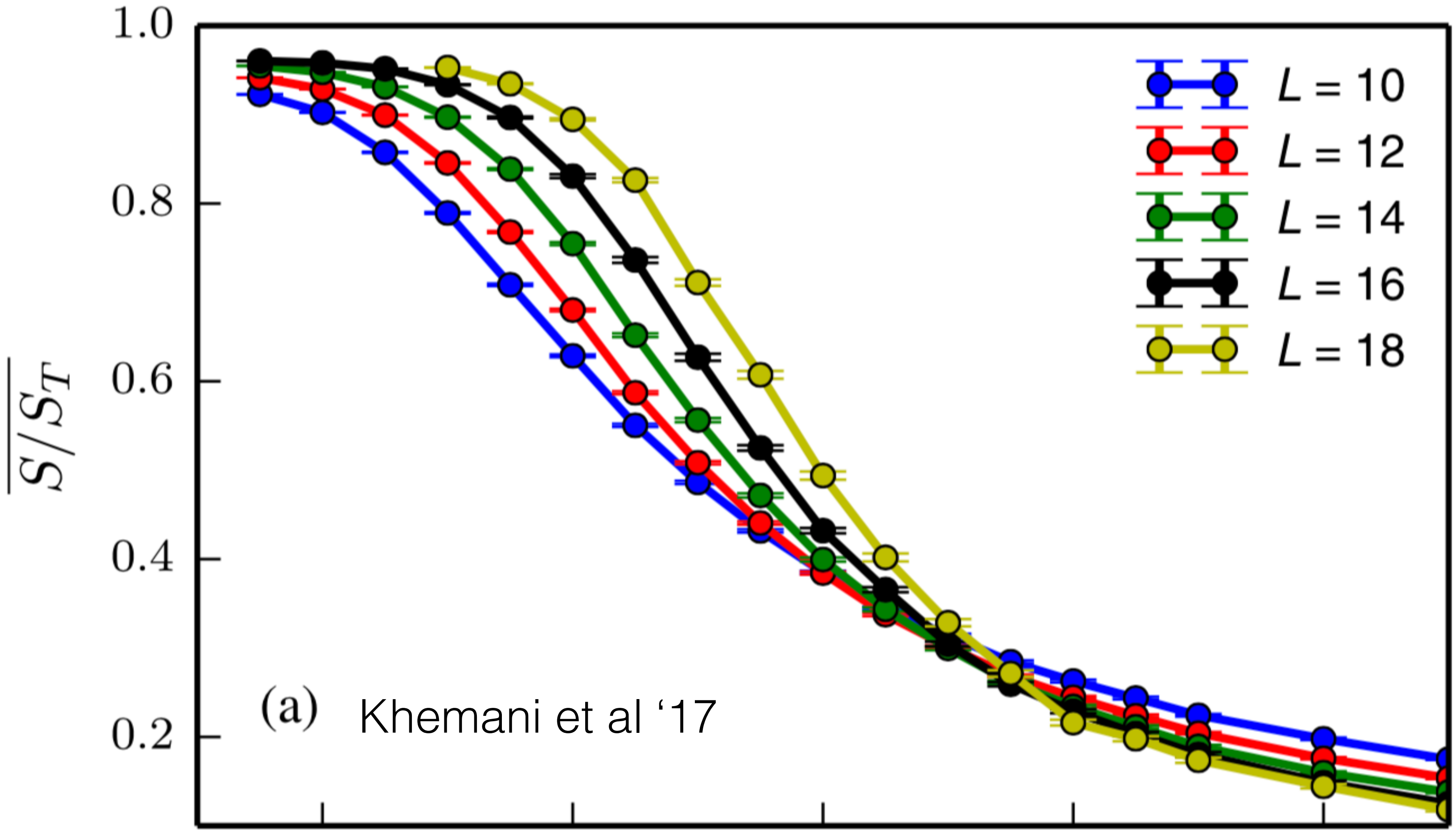
Many-body Localization



- Local conserved quantities (infinitely long local memory of initial state)

- No transport, no thermodynamics, $S_A \propto C$

Eigenstate phases



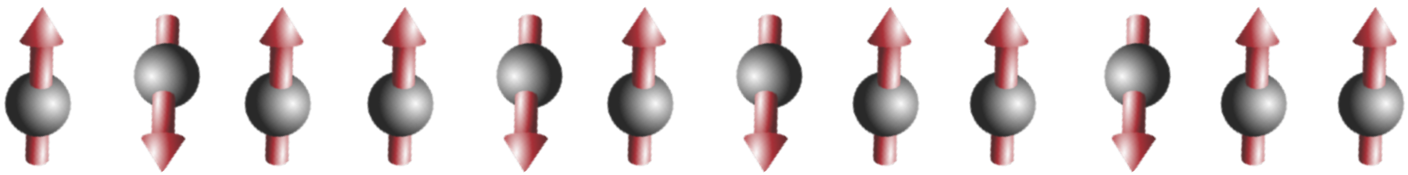
- Numerically many 1D systems appear to undergo a transition from MBL to ETH as disorder/ interactions changes

Constraints

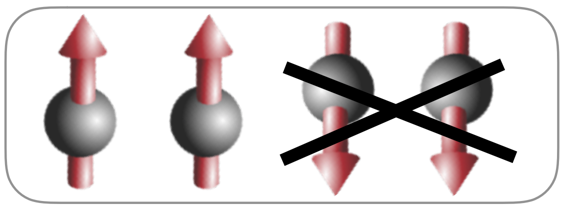
- Constrained systems that you (may) care about
  - Anyon models
  - Dimer models
  - gauge theories
  - Blockaded Rydberg atoms
- Constraints = no local product Hilbert space. Does this affect ETH vs. MBL?

Constrained Hilbert space: example

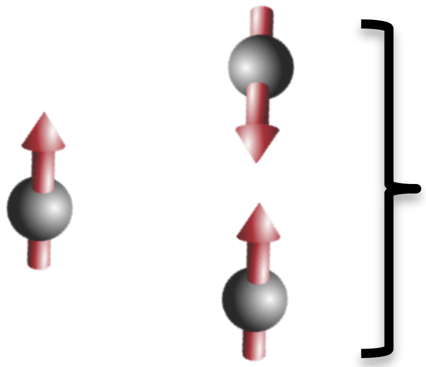
Hilbert space: Ising chain



Constraint: no 2 adjacent spins down



Constraints and locality

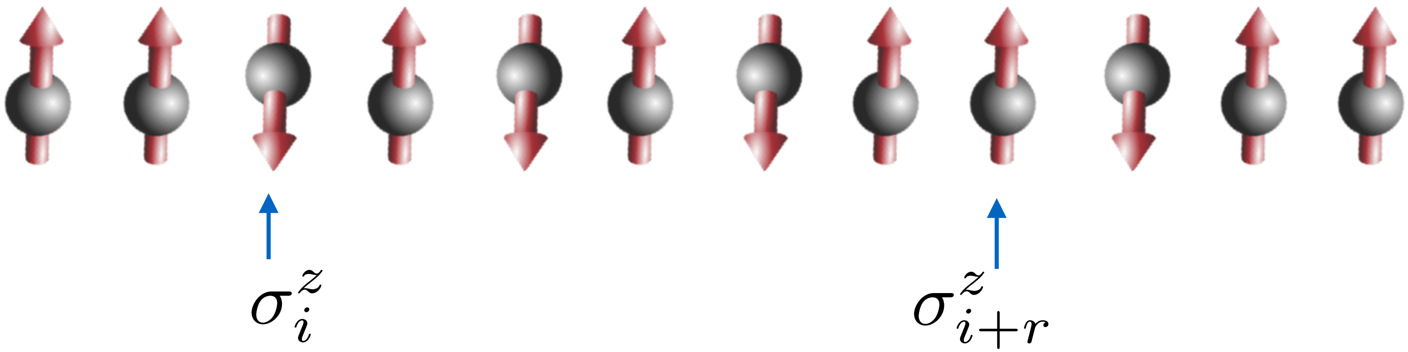


$$\sigma^z_{i+1} = +1 \text{ or } -1$$

$$\sigma^z_{i+1} = +1$$

- Measuring σ^z_i influences σ^z_{i+1}
- No local operators

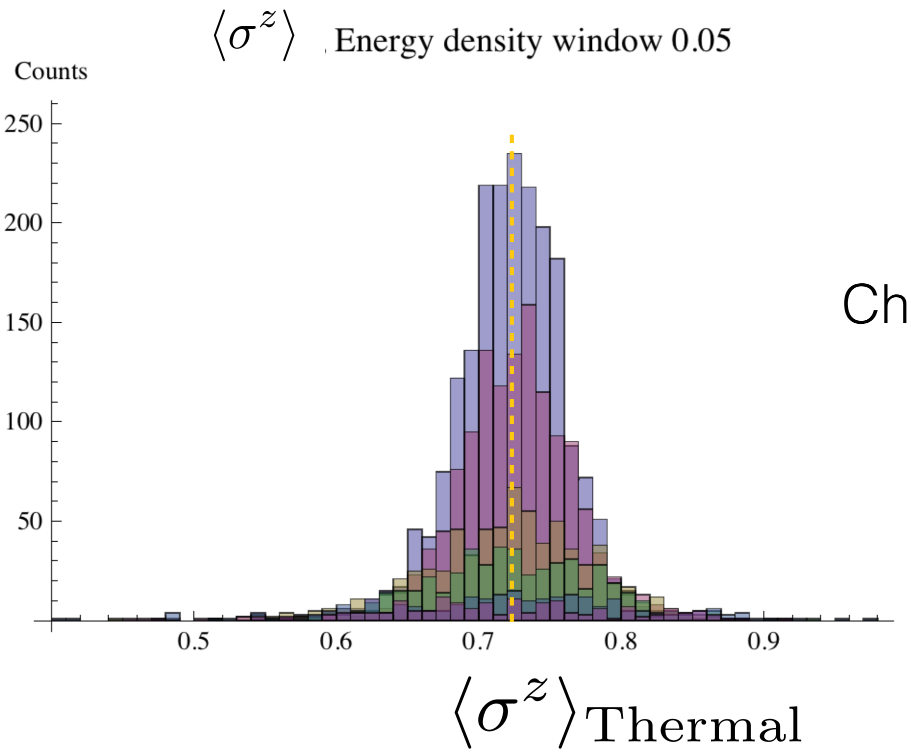
How long-ranged is the entanglement *in the Hilbert space*?



- Effect of first measurement falls off exponentially with inter-bond separation

$$P(\sigma_{i+r}^z = 1 | \sigma_i^z = -1) = P(\sigma_{i+r}^z = 1) P(\sigma_i^z = -1) + \mathcal{O}(d^{-2r})$$

Operators are exponentially local



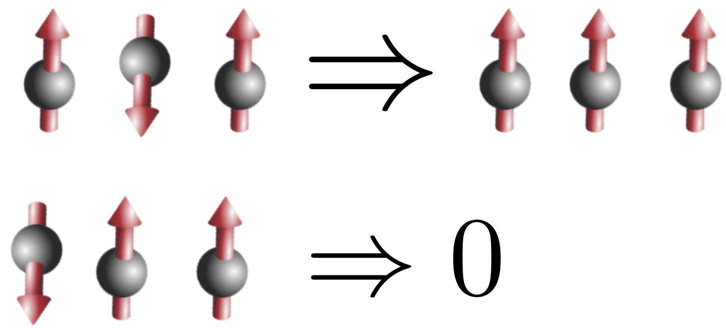
Chandran, Schulz, Burnell '16

- Good enough for ETH!
- MBL?

Constraints and MBL: Model

- Projected Pauli operators

$$\tilde{Z}_i = \sigma_i^z \qquad \tilde{X}_i = P_{\sigma^z_{i-1}=1}\sigma_i^x P_{\sigma^z_{i+1}=1}$$



Constraints and MBL: Model

$$\tilde{Z}_i = \sigma_i^z \qquad \tilde{X}_i = P_{\sigma_{i-1}^z = 1} \sigma_i^x P_{\sigma_{i+1}^z = 1}$$

- (Interacting) Hamiltonian: random fields in X and Z

$$H = \sum_i \left(\overline{g} + g_i\right) \tilde{X}_i + \sum_i h_i \tilde{Z}_i$$

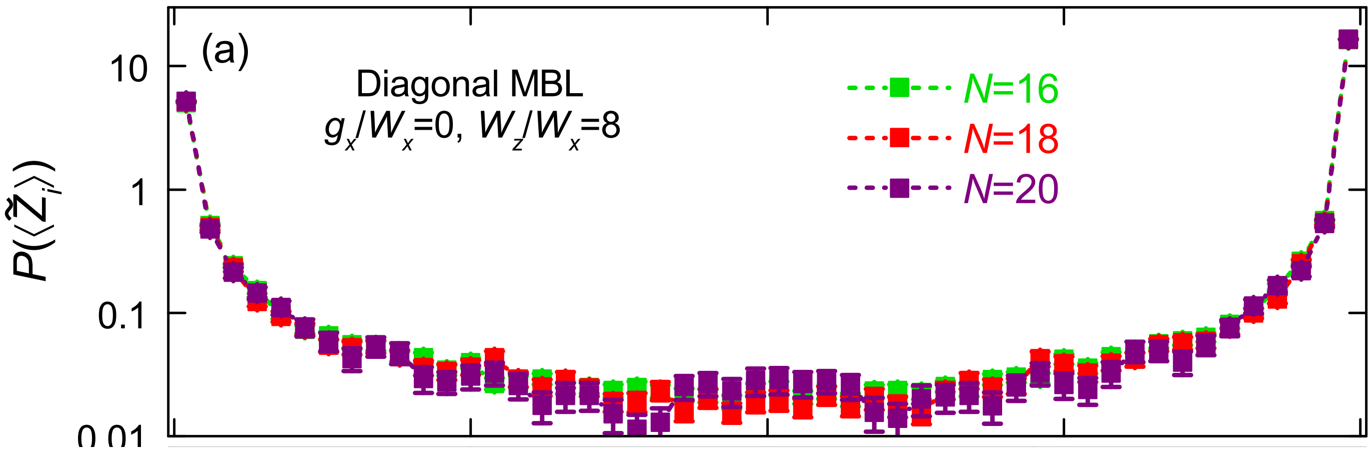
$$g_i \in [-W_x, W_x] \qquad h_i \in [-W_z, W_z]$$

Constrained operators and local conserved quantities (l-bits): diagonal

$$H = \sum_i h_i \tilde{Z}_i$$

$\tilde{Z}_i$ conserved

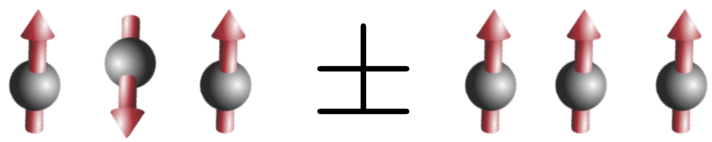
but not independent: $\tilde{Z}_i \tilde{Z}_{i+1} = \tilde{Z}_i + \tilde{Z}_{i+1} - 1$

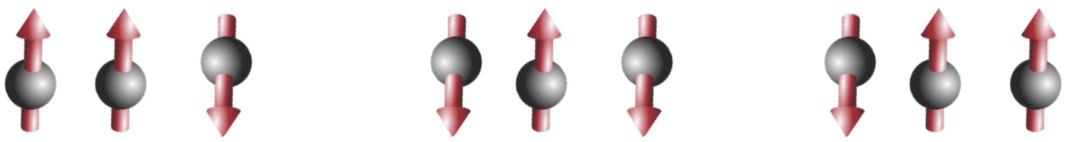


$\langle \tilde{Z}_i \rangle$

Constrained operators and local conserved quantities (l-bits): off-diagonal

$$H = \sum_i g_i \tilde{X}_i$$

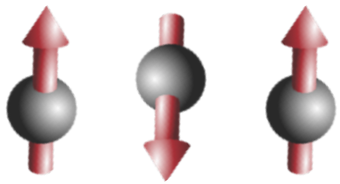
$\frac{1}{\sqrt{2}}($  $)$ $\qquad \tilde{X}_i = \pm 1$



$$\tilde{X}_i = 0$$

- Can't be simultaneous eigenstate of $\tilde{X}_i, \tilde{X}_{i+1}$
- $\{\tilde{X}_i\}$ cannot all be l-bits

Constrained operators and local conserved quantities (l-bits): off-diagonal

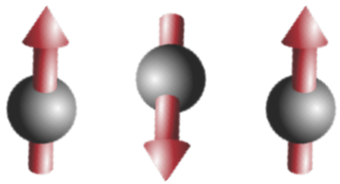


g_1 $\quad 0 \quad$ g_3

$\tilde{Z}_2$ conserved

$\tilde{X}_1\ ,\tilde{Z}_2\ ,\tilde{X}_3$

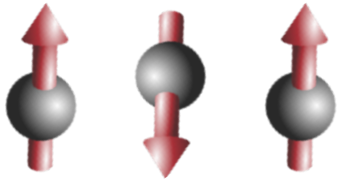
Constrained operators and local conserved quantities (l-bits): off-diagonal



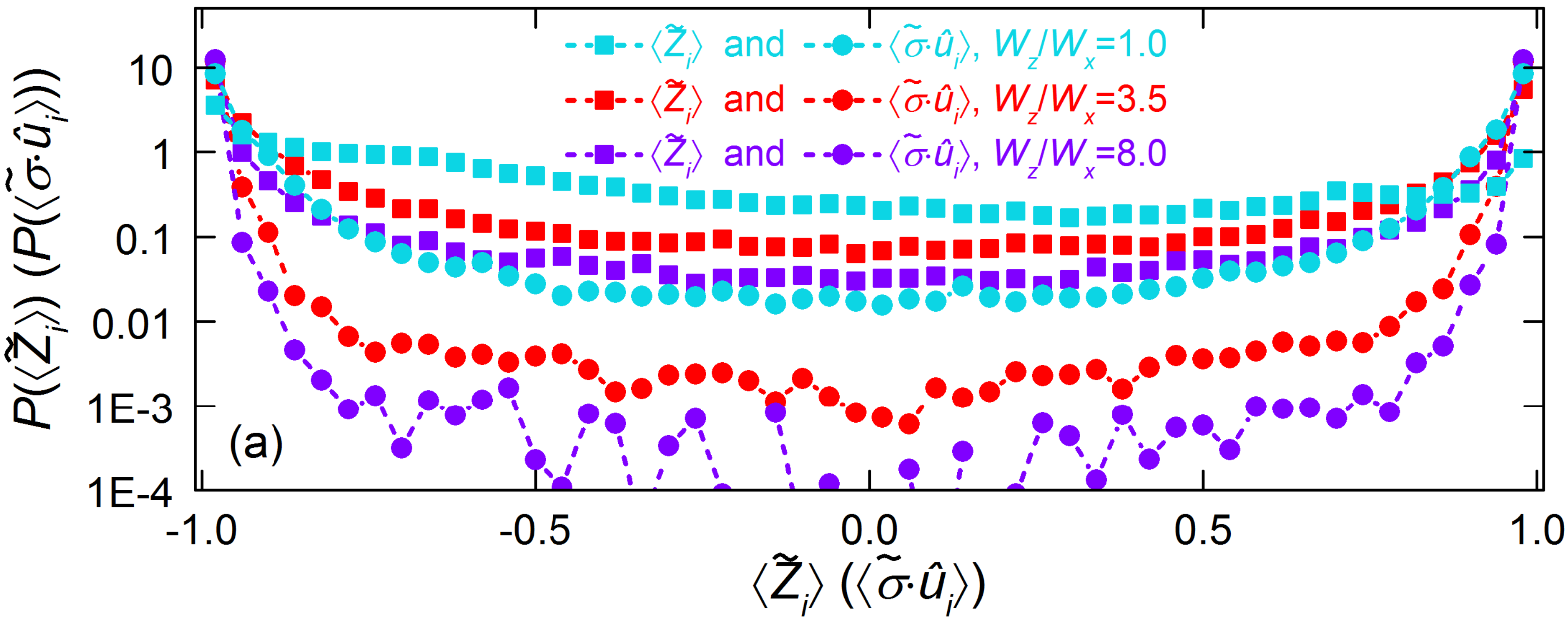
g_1 h_2 g_3 (dominant fields)

$\tilde{X}_1$, $\tilde{Z}_2$, $\tilde{X}_3$:aproximate l-bits

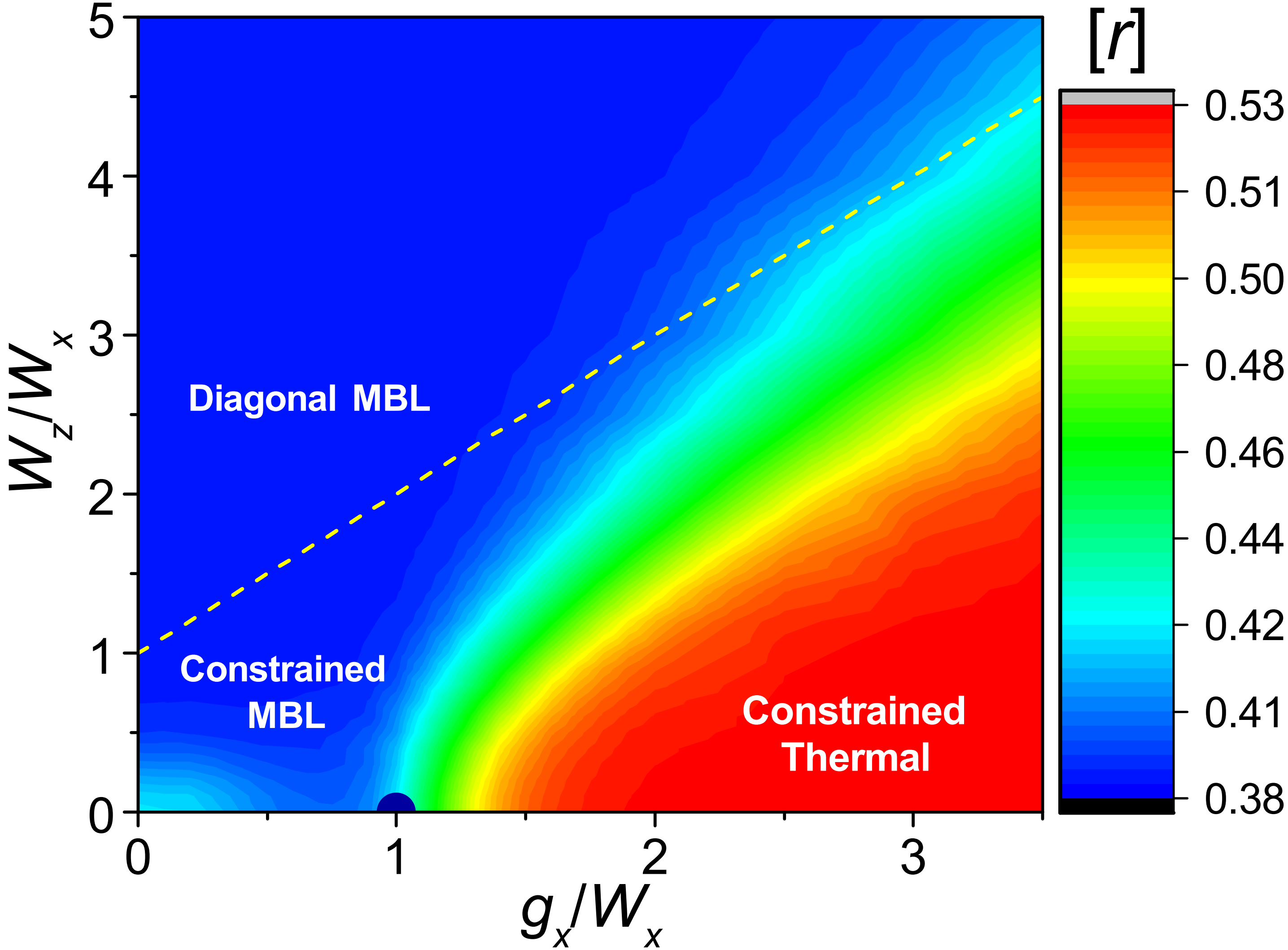
Constrained operators and local conserved quantities (l-bits): off-diagonal



$\tilde{X}_1 \,, \tilde{Z}_2 \,, \tilde{X}_3$:aproximate l-bits

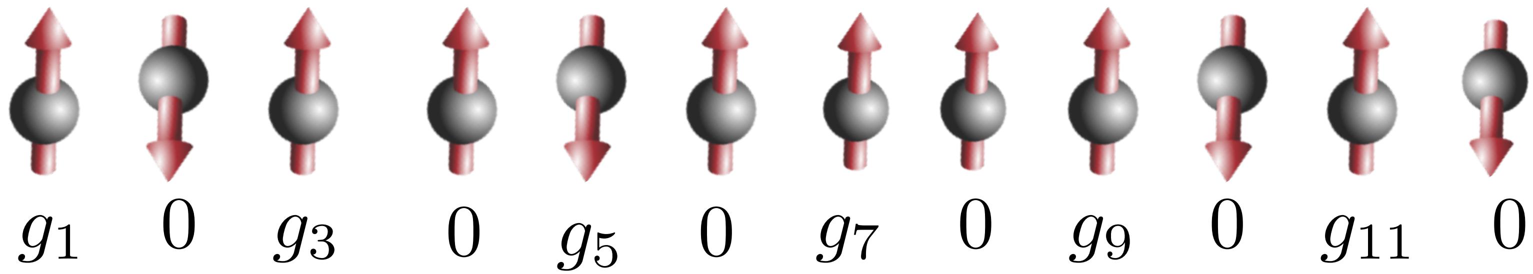


Signatures of localization for $W_x < W_z$

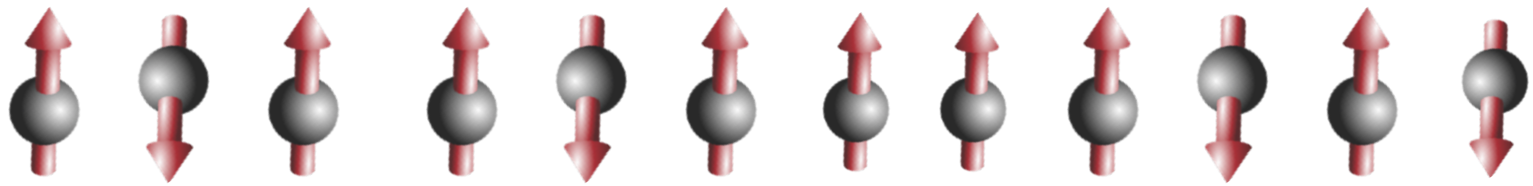


Diagonal MBL
Constrained MBL
Constrained Thermal
W_z/W_x
g_x/W_x
[r]
0.53
0.51
0.50
0.48
0.46
0.44
0.42
0.41
0.38

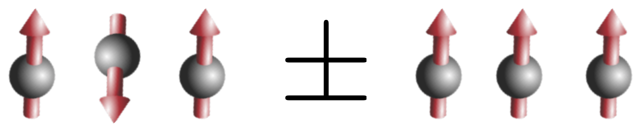
Localization without Z-fields?

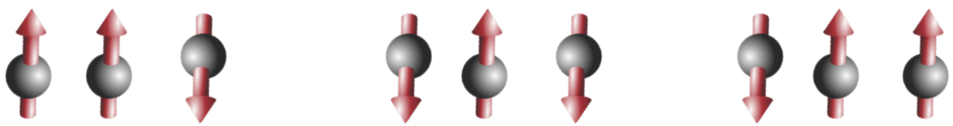


Localization without Z-fields?



$g_1 \quad 0 \quad g_3 \quad 0 \quad g_5 \quad 0 \quad g_7 \quad 0 \quad g_9 \quad 0 \quad g_{11} \quad 0$

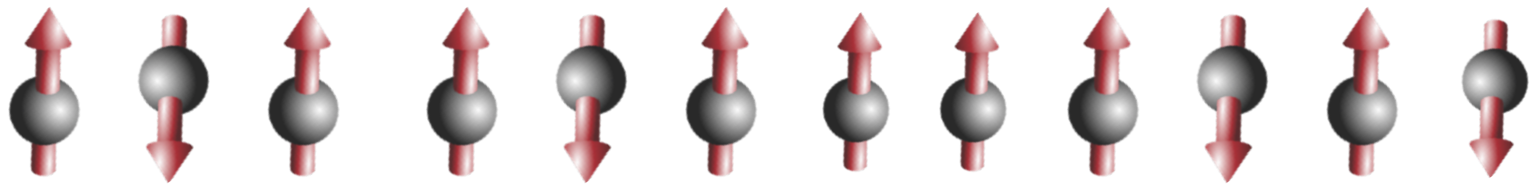
$\frac{1}{\sqrt{2}}($  $)$ $\quad \tilde{X}_i = \pm 1$



$\tilde{X}_i = 0$

- Trouble: degeneracies at 0-energy!

Totally off-diagonal l-bits



- No degeneracy after fixing BC's
- l-bits: $\tilde{X}_i \tilde{X}_{i+1}$ $(i = 1 \text{ Mod } 3)$ stable

Summary

- Constraints = quasi-local operators
- Does not intrinsically destroy thermalization or localization
- Interplay between constraints and disorder leads to complex ``frustrated" eigenstate phase diagrams